

Exercise 54

Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$$\ln x = x - \sqrt{x}, \quad (2, 3)$$

Solution

Bring all terms to one side.

$$\ln x + \sqrt{x} - x = 0, \quad (2, 3)$$

Let $f(x) = \ln x + \sqrt{x} - x$. The logarithmic, root, and polynomial functions are each continuous on their respective domains by Theorem 7. The sum or difference of these functions is also continuous by Theorem 4.

$$f(x) = 0, \quad (2, 3)$$

Find a value of x in the interval $[2, 3]$ so that $f(x)$ is negative, and find a value of x in the interval $[2, 3]$ so that $f(x)$ is positive.

$$f(2) \approx 0.107$$

$$f(3) \approx -0.169$$

$f(x)$ is continuous on the closed interval $[2, 3]$, and $N = 0$ lies between $f(2)$ and $f(3)$. By the Intermediate Value Theorem, then, there exists a number c such that $f(c) = 0$.